Art of Problem Solving logarithm Problems: AIME and AMC 12

1. Suppose that “a” , “b”, and “c” are positive real numbers such that $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$, and $c^{\log_{11} 25} = \sqrt{11}$, find the value of the expression: $a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$ (2009 AIME II)

2. It is given that $\log_6 a + \log_6 b + \log_6 c = 6$, where “a” , “b” , and “c” are positive integers that form an increasing geometric sequence and $b - a$ is the square of an integer. Find $a + b + c$ (AIMe 2002)

3. The expression below can be written as a fraction in lowest terms, simplify it: $\frac{2}{\log_4 2000^6} + \frac{3}{\log_5 2000^6}$

   (2000 AIME)

4. Positive numbers “x” , “y”, and “z” satisfies the equations below:
   $xyz = 10^{81}$ and $(\log x)(\log yz) + (\log y)(\log z) = 468$, find the value of
   $\sqrt{(\log x)^2 + (\log y)^2 + (\log z)^2}$ (2010 aime)

5. Given the system of equations below and that it has two solutions (x_1, y_1, z_1) and (x_2, y_2, z_2) . Find $y_1 + y_2$

   $\log(2000xy) - (\log x)(\log y) = 4$

   $\log(2yz) - (\log y)(\log z) = 1$

   $\log(zx) - (\log z)(\log x) = 0$

6. The sequence a1, a2, a3……. Is geometric with a1=a and common ratio 'r", where "a" and "r" are positive integers. Given that $\log_8 a_1 + \log_8 a_2 + ... + \log_8 a_{12} = 2006$, find the number of possible ordered pairs (a,r)? (2006 aime)

7. Suppose "A" and "B" are positive real numbers for which $\log_A B = \log_B A$. If neither "A" nor "B" is 1 and $A \neq B$, find the value of AB .

8. Given that $\log_3 2 \approx 0.631$, find the smallest positive integer "a" such that $3^a > 2^{102}$
{Hint: Show that $\log_3 2^{102} = 102 \log_3 2$ }

9. Given that $\log_{10} \sin x + \log_{10} \cos x = -1$ and that $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, find the value of "n"? (2003 AIME)

10. How many real numbers "x" satisfy the equation: $\frac{1}{5}\log_2 x = \sin(5\pi x)$? (AIME 1991)

11. Find the last three digits of the product of the positive roots of the equation below:

$\sqrt{1995}x\log_{1995} x = x^2$ (1995 AIME)

12. Suppose "x' is in the interval $[0,\frac{\pi}{2}]$ and $\log_{24\sin x}(24\cos x)=\frac{3}{2}$, find the value of $24\cot^2 x$ (AIME 2011)

13. The solutions to the system of equations below are (x_1, y_1) and (x_2, y_2). Find the value of

$\log_{30}(x_1 \times y_1 \times x_2 \times y_2)$ (2002 AIME)

$\log_{225} x + \log_{64} y = 4$

$\log_x 225 - \log_y 64 = 1$

14. . Let "x", "y", and "z" be positive real numbers that satisfy the equation below. If the value of xy^5z can be expressed in the form $\frac{1}{2^{p/q}}$, where "p" and "q" are relative prime positive integers, find "p+q".

$2\log_x(2y) = 2\log_{2x}(4z) = \log_{2x^4}(8yz) \neq 0$ (2012 AIME)

15. For certain pairs (m,n) of positive integers with $m \geq n$ there are exactly 50 distinct positive integers "k" such that $\left|\log m - \log k\right| < \log n$. Find the sum of all possible values of the product mn (2009 AIME)

16. The increasing geometric sequence $x_o, x_1, x_2, \ldots\ldots$ consists entirely of integral powers of 3. Given that $\sum_{n=0}^{7} \log_3(x_n) = 308$ and $56 \leq \log_3\left(\sum_{n=0}^{7} x_n\right) \leq 57$, find the value of $\log_3(x_{14})$